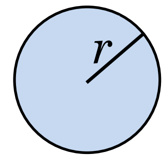
CIRCLES – ALGEBRA

❑ REVIEW OF CIRCLE FORMULAS



If r is the radius of a **circle**, then its *diameter* is **2*r***, its *circumference* is **2π*r***, and its *area* is $\boldsymbol{\pi r^2}$.

Recall from the Order of Operations that in the formula πr^2, we square the r first, and then multiply by π.

π is defined as the ratio of the circumference of a circle to its diameter:

$$\pi = \frac{C}{d}$$

Homework

1. The radius of a circle is 10. Find the diameter, the circumference, and the area in exact form (that means leave π as π).

2. The radius of a circle is 1. Find the diameter, the circumference, and the area.

3. The radius of a circle is $\frac{1}{2}$. Find the diameter, the circumference, and the area.

❑ DERIVING THE CIRCUMFERENCE FORMULA: $C = 2\pi r$

The first problem we will solve using our algebra skills is to derive the formula $C = 2\pi r$ for the circumference of a circle whose radius is r.

We begin with the **definition** of π, namely that π is the ratio of the circumference of any circle to its diameter:

$$\pi = \frac{C}{d}$$

Multiplying each side of this equation by d gives:

$$\pi[d] = \frac{C}{\not{d}}[\not{d}]$$

Simplifying gives

$$\pi d = C$$

Now we turn the equation around and also change d into $2r$, since the diameter is twice the radius:

$$C = \pi(2r)$$

Applying the fact that multiplication is an associative operation, we can "shift" the parentheses to associate the π and the 2:

$$C = (\pi 2)r$$

Since multiplication is also a commutative operation, we can switch the π and the 2 to get:

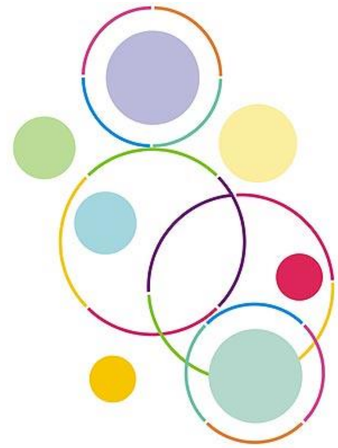
$$C = (2\pi)r$$

Again, since multiplication is an associative operation, the parentheses are redundant (unnecessary) and we can remove them. Our final formula:

$$\boxed{\boldsymbol{C = 2\pi r}}$$

❑ USING ALGEBRA TO SOLVE CIRCLE PROBLEMS

<u>EXAMPLE 1:</u> **The diameter of a circle is 12. Find the area.**



<u>Solution:</u> The formula for the area of a circle, $A = \pi r^2$, requires that we have the radius, which was not explicitly given to us in the problem. But the diameter of the circle <u>was</u> given, so we can use this diameter to find the radius, and then use the radius to find the area:

given in the problem

$$r = \frac{d}{2} = \frac{12}{2} = \mathbf{6}$$

Now we know that $r = 6$, so the area is calculated as follows:

$$A = \pi r^2 = \pi \cdot 6^2 = \boxed{36\pi}$$

<u>EXAMPLE 2:</u> **Find the circumference of a circle whose diameter is 9.**

<u>Solution:</u> The process is analogous to the previous problem.

$r = \frac{d}{2} = \mathbf{\frac{9}{2}}$, and so

$$C = 2\pi r = 2\pi\left(\frac{9}{2}\right) = \cancel{2}\pi\left(\frac{9}{\cancel{2}}\right) = \boxed{9\pi}$$

<u>EXAMPLE 3:</u> **The circumference of a circle is 24π. Find the radius.**

<u>Solution:</u> The circumference of a circle is given by $C = 2\pi r$. Also, the circumference is given to be 24π. So if we set the

circumference formula ($2\pi r$) equal to the given circumference (24π), we should be able to solve for r:

$$2\pi r = 24\pi \qquad (C = 2\pi r)$$

$$\Rightarrow \frac{2\pi r}{2\pi} = \frac{24\pi}{2\pi} \qquad \text{(divide each side by } 2\pi\text{)}$$

$$\Rightarrow \frac{\cancel{2\pi} r}{\cancel{2\pi}} = \frac{\overset{12}{\cancel{24}}\cancel{\pi}}{\underset{1}{\cancel{2}}\cancel{\pi}} \qquad \text{(divide out common factors)}$$

$$\Rightarrow \boxed{r = 12} \qquad \text{(and we've isolated the } r\text{)}$$

EXAMPLE 4: **Find the radius of a circle if it's known that its area is 121π.**

Solution: We set the area formula (πr^2) equal to the given area (121π), and solving for r will be as easy as π.

$$\pi r^2 = 121\pi \Rightarrow \frac{\cancel{\pi} r^2}{\cancel{\pi}} = \frac{121\cancel{\pi}}{\cancel{\pi}} \Rightarrow r^2 = 121$$

Using a calculator, or better yet, your brain, we see that $\boxed{r = 11}$

Note: The equation $r^2 = 121$ actually has two solutions, 11 and -11. But we discard the negative solution since the radius of a circle must be a positive number.

Homework

4. Find the circumference and the area of the circle whose diameter is given:

   a. $d = 10$ b. $d = 13$ c. $d = 2$ d. $d = 200$

5. Find the radius of the circle given its circumference or area:

a. $C = 30\pi$ b. $C = 19\pi$ c. $A = 100\pi$ d. $A = \pi$

EXAMPLE 5: **The circumference of a circle is 4.6π. Find the area.**

Solution: First ask yourself, What do I need to find the area? The radius, of course. But was the radius given to us? No, but the circumference was, so we can use the circumference to find the radius, and then use that radius to find the area.

Step 1: Use the circumference to find the radius

The circumference is given to be 4.6π.

$\Rightarrow \quad 2\pi r = 4.6\pi$ (circumference = $2\pi r$)

$\Rightarrow \quad \dfrac{2\pi r}{2\pi} = \dfrac{4.6\pi}{2\pi}$ (divide each side by 2π)

$\Rightarrow \quad r = 2.3$ (simplify each side)

Step 2: Use the radius to find the area

$$A = \pi r^2 = \pi(2.3)^2 = \boxed{5.29\pi}$$

EXAMPLE 6: **The area of a circle is 2.25π. Find the circumference.**

Solution: Reversing the logic in the previous problem, we use the given area to calculate the radius, and then use that radius to find the circumference:

$$\pi r^2 = 2.25\pi \Rightarrow \frac{\pi r^2}{\pi} = \frac{2.25\pi}{\pi} \Rightarrow r^2 = 2.25 \Rightarrow r = \mathbf{1.5}$$

It thus follows that $C = 2\pi r = 2\pi(1.5) = 2(1.5)\pi = \boxed{3\pi}$

Homework

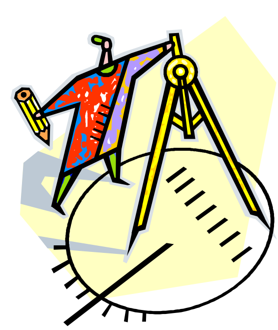
6. Find the area of the circle with the given circumference:

   a. $C = 10\pi$ b. $C = 2\pi$ c. $C = \pi$ d. $C = 5\pi$

7. Find the circumference of the circle with the given area:

   a. $A = 25\pi$ b. $A = \pi$ c. $A = 81\pi$ d. $A = 225\pi$

8. a. Find the circumference of a circle with diameter 2.
   b. Find the area of a circle with diameter 30.
   c. Find the radius of a circle with circumference 38π.
   d. Find the diameter of a circle with circumference 8π.
   e. Find the radius of a circle with area 576π.
   f. Find the diameter of a circle with area 400π.
   g. Find the area of a circle with circumference 2π.
   h. Find the circumference of a circle with area 16π.
   i. Find the circumference of a circle with diameter 28.
   j. Find the area of a circle with diameter 28.
   k. Find the radius of a circle with circumference 34π.
   l. Find the diameter of a circle with circumference 20π.
   m. Find the radius of a circle with area 529π.
   n. Find the diameter of a circle with area 81π.
   o. Find the area of a circle with circumference 18π.
   p. Find the circumference of a circle with area 169π.



Review Problems

9. The circumference of a circle is 20π. Find the area.

10. The area of a circle is 49π. Find the circumference.

11. True/False:

    a. If the diameter of a circle is 12, its area is 144π.

    b. If the diameter of a circle is 20, its circumference is 20π.

    c. If the circumference of a circle is 20π, its area is 100π.

    d. If the area of a circle is 49π, its diameter is 14.

    e. If the radius of a circle is 11, its area is 121.

    f. If the area of a circle is 36π, its circumference is 36π.

Solutions

1. $d = 20$; $C = 20\pi$; $A = 100\pi$

2. $d = 2$; $C = 2\pi$; $A = \pi$

3. $d = 1$; $C = \pi$; $A = \frac{1}{4}\pi$

4. a. $C = 10\pi$; $A = 25\pi$ b. $C = 13\pi$; $A = 42.25\pi$
c. $C = 2\pi$; $A = \pi$ d. $C = 200\pi$; $A = 10{,}000\pi$

5. a. $r = 15$ b. $r = 9.5$ c. $r = 10$ d. $r = 1$

6. a. $A = 25\pi$ b. $A = \pi$ c. $A = 0.25\pi$ d. $A = 6.25\pi$

7. a. $C = 10\pi$ b. $C = 2\pi$ c. $C = 18\pi$ d. $C = 30\pi$

8. a. $C = 2\pi$ b. $A = 225\pi$ c. $r = 19$ d. $d = 8$

e. $r = 24$ f. $d = 40$ g. $A = \pi$ h. $C = 8\pi$
i. $C = 28\pi$ j. $A = 196\pi$ k. $r = 17$ l. $d = 20$
m. $r = 23$ n. $d = 18$ o. $A = 81\pi$ p. $C = 26\pi$

9. 100π **10**. 14π

11. a. F b. T c. T d. T e. F f. F

❑ *To ∞ and Beyond*

A. Find a circle whose area is equal to its circumference (even though the units must be different).

B. Find the radius of a circle whose circumference is 20.

C. Find the radius of a circle whose area is 10.

D. By what factor must the radius of a circle be increased to increase the circumference by a factor of 4?

E. By what factor must the radius of a circle be increased to increase the area by a factor of 9?

"The wisest mind has something yet to learn."

George Santayana (1863 - 1952)